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RESEARCH MEMORANDUM

THEORETICAL PERFORMANCE OF LIQUID HYDROGEN AND LIQUID FLUORINE AS A ROCKET PROPELLANT

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NATIONAL ADVISORY COMMITTEE
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THEORETICAL PERFORMANCE OF LIQUID HYDROGEN AND LIQUID FLUORINE AS A ROCKET PROPELLANT

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SUMMARY

Theoretical values of performance parameters for liquid hydrogen and liquid fluorine as a rocket propellant were calculated on the assumption of equilibrium composition during the expansion process for a wide range of fuel-oxidant and expansion ratios. The parameters included were specific impulse, combustion-chamber temperature, nozzle-exit temperature, equilibrium composition, mean molecular weight, characteristic velocity, coefficient of thrust, ratio of nozzle-exit area to throat area, specific heat at constant pressure, coefficient of viscosity, and coefficient of thermal conductivity.

The maximum value of specific impulse was 364.6 pound-seconds per pound for a chamber pressure of 300 pounds per square inch absolute (20.41 atm) and an exit pressure of 1 atmosphere.

INTRODUCTION

Liquid hydrogen and liquid fluorine are of interest as a rocket propellant because of their extremely high performance. Extensive data exist in the literature on their availability, cost, and physical, chemical, and handling properties.

The performance of liquid hydrogen and liquid fluorine has been reported in the literature by a number of organizations such as Jet Propulsion Laboratory, California Institute of Technology; The RAND Corp.; North American Aviation, Inc.; and NACA. Additional performance calculations for this propellant were made at the NACA Lewis laboratory as part of a series of calculations on propellants containing the chemical elements hydrogen, fluorine, and nitrogen to provide a comparison with the performance of other propellants based on the same thermodynamic data and computed to the same degree of accuracy, and to provide several parameters not previously published.

Data were calculated on the basis of equilibrium composition during expansion and cover a wide range of fuel-oxidant and expansion ratios. The performance parameters included are specific impulse, combustion-chamber temperature, nozzle-exit temperature, equilibrium composition,

mean molecular weight, characteristic velocity, coefficient of thrust, ratio of nozzle-exit area to throat area, specific heat at constant pressure, coefficient of viscosity, and coefficient of thermal conductivity.

In order to compare data based on the assumptions of equilibrium and frozen composition during the expansion process, several additional calculations were made in which frozen composition was assumed.

SYMBOLS

The following symbols are used in this report:

A	number of equivalent formulas (a function of pressure and molecular weight; see ref. 1)
a	local velocity of sound, ft/sec
C_F	coefficient of thrust
c_p	specific heat at constant pressure, cal/(g)($^\circ$K)
c_p/c_v	ratio of specific heats
c^*	characteristic velocity, ft/sec
D_A	$\left(\frac{\partial \log A}{\partial \log T}\right)_s$
D_i	$\left(\frac{\partial \log p_i}{\partial \log T}\right)_s$
$f_1, f_2, \ldots f_5$	functions
g	acceleration due to gravity, 32.174 ft/sec^2
H_T°	sum of sensible enthalpy and chemical energy, cal/mole
h	sum of sensible enthalpy and chemical energy per unit weight, $\frac{\sum_i n_i \left(H_T^\circ\right)_i}{nM}$, cal/g
I	specific impulse, lb-sec/lb

k	coefficient of thermal conductivity, cal/(sec)(cm)(^{o}K)
L	mean free path, cm
M	molecular weight, g/mole
n	number of moles; exponent
P	pressure
p	partial pressure
R	universal gas constant (consistent units)
r	equivalence ratio, ratio of number of fluorine atoms to hydrogen atoms
r_o	collision diameter for low-energy head-on collision between two molecules, A
S	nozzle area, sq ft
T	temperature, ^{o}K
$V/\left[W^{(2)}(2)\right]$	a function of kT/ϵ where k is Boltzmann's constant and ϵ is the energy difference between separated molecules and molecules in the configuration for which they have the maximum energy of attraction
$\overline{v}$	mean molecular speed, cm/sec, equal to $0.921\sqrt{3RT/M}$
w	rate of flow, lb/sec
γ_s	$\left(\frac{\partial \log P}{\partial \log \rho}\right)_s$
μ	coefficient of viscosity, g/(cm)(sec) = poise
ρ	density, g/cm^3
σ	molecular diameter for viscosity and heat conduction, cm

Subscripts:

c	combustion chamber
e	nozzle exit

i	product of combustion
max	maximum
o	conditions at 0° K, assuming recombination is complete
s	constant entropy
T	temperature
t	nozzle throat
x	any point in nozzle

CALCULATION OF PERFORMANCE

Equilibrium composition, combustion temperature, velocity of sound, and specific heat at constant pressure were calculated by the method described in reference 1 with modifications to adapt it for use with automatic computing machines. The Bell computer at the NACA Langley laboratory was operated with seven significant figures and was used to compute combustion and exit conditions. The successive approximation process used to obtain the desired values of the assigned parameters (mass balance and pressure or entropy balance) was continued until six-figure accuracy was achieved. The IBM Card Programmed Electronic Calculator at the NACA Lewis laboratory, which was used for all interpolations and calculation of transport properties, was operated with numbers in floating-point notation and eight significant figures.

Assumptions

The calculations were based on the following usual assumptions: perfect gas law, adiabatic combustion at constant pressure, isentropic expansion, no friction, homogeneous mixing, and one-dimensional flow. The products of combustion were assumed to be ideal gases and included the following substances: hydrogen H_2, hydrogen fluoride HF, fluorine F_2, atomic hydrogen H, and atomic fluorine F.

Thermodynamic Data

The thermodynamic data used in the calculations were taken from reference 1 which selected the lower value of 35.6 kilocalories per mole for the dissociation energy of F_2 (see ref. 2). Physical and thermochemical properties of the propellants were taken from references 1 to 5 and are given in table I.

Transport Properties

Viscosity and thermal conductivity data are needed for heat-transfer calculations; however, accurate data for gases at high temperatures are unavailable in the literature. In order to obtain a first approximation to the transport properties of mixtures of combustion gases for propellants containing fluorine, hydrogen, and nitrogen, coefficients of viscosity for the individual components were estimated as described in the following paragraphs and are given in table II:

H_2 and N_2. - Data for H_2 and N_2 were calculated by the method of reference 6, which gives the following expression for the coefficient of viscosity

$$\mu \times 10^7 = 266.93 \, (MT)^{\frac{1}{2}} \, (r_o)^{-2} \, V/\left[W^{(2)}(2)\right] \tag{1}$$

The values of r_o and ϵ and tables of the functions $V/\left[W^{(2)}(2)\right]$ used in the calculations were taken from reference 6.

F, H, and N. - Data for F, H, and N were calculated by the method of reference 7 which relates coefficient of viscosity and temperature according to the equation

$$\mu_T = \mu_{288} \, (T/288)^n \tag{2}$$

The following equation from reference 8 was used to compute the coefficient of viscosity at 288° K

$$\mu = 0.499 \, \rho \bar{v} L \tag{3}$$

The exponent n may be estimated from figure 1 (taken from ref. 7) which is a plot of n versus σ for a number of gases.

The values of μ_{288} calculated from equation (3) and of n estimated from figure 1 are as follows:

Gas	μ_{288}, poise × 10^7	n
F	2751	0.695
H	802	.663
N	1916	.734

HF. - No experimental values for the viscosity of HF were found in the literature. The experimental values of μ_{288} for the other hydrogen halides were found to differ from the values obtained from equation (3). Therefore, the value of μ_{288} for HF obtained from equation (3) was corrected by a similar difference resulting in a value of 0.0001603 poise.

Because HF is a polar molecule, the value of the exponent n for HF would be expected to lie above the curve of figure 1. A value 0.984 for n was estimated from the values of the exponents of the other hydrogen halides.

Inasmuch as the value of viscosity of HF at high temperatures is very uncertain and HF is present in very considerable quantities in the combustion products, caution should be exercised in the use of these data. However, the data tabulated are believed to be sufficiently accurate for most engineering purposes until better data become available.

F_2. - Viscosity data for F_2 were not estimated since the amount of F_2 which exists as a reaction product is negligible.

Method of Calculation

Procedure for combustion conditions. - For each of 11 equivalence ratios, temperature, equilibrium composition, enthalpy, mean molecular weight, derivative of the logarithm of pressure with respect to the logarithm of density at constant entropy γ_s, specific heat at constant pressure, coefficient of viscosity, coefficient of thermal conductivity, and entropy of the combustion products were computed at a combustion pressure of 300 pounds per square inch absolute (20.41 atm).

The function $\left(\frac{\partial \log P}{\partial \log \rho}\right)_s = \gamma_s$ was used in the computation of throat conditions, since

$$a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s = \left(\frac{\partial \log P}{\partial \log \rho}\right)_s \frac{P}{\rho} = \gamma_s \, R\frac{T}{M} \tag{4}$$

The derivative γ_s is equal to the ratio of specific heats c_p/c_v only when the molecular weight is constant. In the nomenclature of reference 1,

$$\gamma_s = \frac{\sum_i p_i D_i}{P(D_A - 1)} \tag{5}$$

where

$$D_i = \left(\frac{\partial \log p_i}{\partial \log T}\right)_s$$

and

$$D_A = \left(\frac{\partial \log A}{\partial \log T}\right)_s$$

The numerical values of D_i and D_A were computed by the method given in reference 1 and were used to calculate the value of γ_s.

Procedure for exit conditions. - Equilibrium composition, mean molecular weight, pressure, derivative of the logarithm of pressure with respect to the logarithm of density at constant entropy γ_s, enthalpy of the products of combustion, specific heat at constant pressure, coefficient of viscosity and coefficient of thermal conductivity were computed for each equivalence ratio by assuming isentropic expansion for four assigned exit temperatures selected to cover the exit pressure range from the nozzle-throat pressure to about 0.1 atmosphere.

Interpolation formulas. - Throat parameters and exit parameters corresponding to altitudes of 0, 10,000, 20,000, 30,000, 40,000, and 50,000 feet were interpolated by means of cubic equations between each pair of the assigned exit temperatures. The coefficients of the cubic equations were determined from the values of the following functions and their first derivatives at each pair of the assigned exit temperatures.

$$f_1 = \ln\left(\frac{h}{R} + \frac{\gamma_s T}{2M} - \frac{h_o}{R}\right)$$

$$f_2 = h/R$$

$$f_3 = \ln T$$

$$f_4 = \ln M$$

$$f_5 = \ln P$$

$$\frac{df_1}{df_5} = \frac{T}{2Mf_1}\left(\gamma_s + 1 + \frac{d\gamma_s}{df_5}\right)$$

$$\frac{df_2}{df_5} = \frac{T}{M}$$

$$\frac{df_3}{df_5} = \frac{1}{\gamma_s(D_A - 1)}$$

$$\frac{df_4}{df_5} = \frac{D_A}{\gamma_s(D_A - 1)} - 1$$

(The value of $d\gamma_s/df_5$ was found by a numerical method.)

The pressure at the throat was found by interpolating f_5 as a function of f_1 for the point $f_1 = \ln (h_c/R - h_o/R)$, at which the velocity of flow equals the velocity of sound. The values of the remaining functions were interpolated as functions of f_5, for the desired pressures.

The errors due to interpolation were checked for several cases. The values presented for all performance parameters appear to be correctly interpolated to one or two units in the last place tabulated.

Formulas

The formulas used in computing the various performance parameters are given in the following paragraphs:

Specific impulse. - Specific impulse was calculated from the difference in enthalpy between the combustion chamber and the nozzle exit by the following equation derived from the energy equation

$$I = 294.98 \sqrt{\frac{h_c - h_e}{1000}} \tag{6}$$

Throat area per unit flow rate. - For equilibrium composition during expansion, the throat area per unit flow rate was obtained from the continuity equation and with pressure in atmospheres becomes

$$S_t/w = \frac{1.3144\ T_t}{P_t\ M_t\ a} \tag{7}$$

Characteristic velocity. - The equation for characteristic velocity for a combustion pressure of 300 pounds per square inch absolute becomes

$$c^* = gP_c\ S_t/w = 1.3899\times10^6\ S_t/w \tag{8}$$

Coefficient of thrust. - The coefficient of thrust was obtained from the defining equation

$$C_F = Ig/c^* = 32.174\ I/c^* \tag{9}$$

Area ratios. - In order to calculate ratio of nozzle-exit area to throat area S_e/S_t, values of the nozzle-exit area per unit flow rate were first obtained from the equation:

$$S_e/w = \frac{0.040853\ T_e}{P_e\ M_e\ I} \tag{10}$$

where P_e is in atmospheres.

Coefficient of viscosity. - The coefficient of viscosity for a mixture of combustion products was obtained by averaging the viscosities of the individual components according to the equation

$$\mu = \frac{PM}{\sum_i \frac{p_i}{(\mu_i/M_i)}} \tag{11}$$

Several other methods for obtaining viscosities of mixtures are given in the literature (refs. 9 and 10). A check made at several points between the values of coefficient of viscosity obtained by equation (11) and by the other more laborious methods showed that the differences resulting were insignificant compared to the uncertainties in the viscosity data for the individual components.

Coefficient of thermal conductivity. - The coefficient of thermal conductivity for a mixture of combustion products was obtained from the values of specific heat at constant pressure and coefficient of viscosity of the mixture according to the Euken equation written in the form

$$k = \mu\left(c_p + \frac{5}{4}\frac{R}{M}\right) \tag{12}$$

Specific heat at constant pressure. - The specific heat at constant pressure is defined as follows:

$$c_p = \left(\frac{\partial h}{\partial T}\right)_P = \left(\frac{\partial \sum_i \frac{n_i (H_T^\circ)_i}{nM}}{\partial T}\right)_P \tag{13}$$

In the nomenclature of reference 1

$$c_p = \frac{1}{nMT}\left[T \sum_i n_i (C_p^\circ)_i + \sum_i n_i (H_T^\circ)_i \, Y_i + \sum_i n_i (H_T^\circ)_i \, Y_A\right] \tag{14}$$

where

$$Y_i = \left(\frac{\partial \log n_i}{\partial \log T}\right)_P$$

$$Y_A = \left(\frac{\partial \log A}{\partial \log T}\right)_P$$

and C_p° is the molar specific heat at constant pressure. The numerical values of the partial derivatives Y_i and Y_A were computed by the method given in reference 1 and were used to compute the value of c_p.

Specific heat and specific heat ratio for frozen composition. - In the case of frozen composition, the values of D_i and D_A are equal to $\gamma/\gamma-1$ and the values of Y_i and Y_A are equal to zero. Equation (13) therefore reduces to the familiar form

$$(c_p)_{frozen} = \sum_i \frac{n_i (C_p^\circ)_i}{nM} \tag{15}$$

and equation (5) reduces to $\gamma_s = \gamma$ where

$$\gamma = \frac{(c_p)_{frozen}}{(c_p)_{frozen} - R/M} \tag{16}$$

THEORETICAL PERFORMANCE

The calculated values of the various performance parameters for a combustion pressure of 300 pounds per square inch absolute and at exit pressures corresponding to altitudes of 0, 10,000, 20,000, 30,000, 40,000, and 50,000 feet are given in table III for 11 equivalence ratios. The values of pressure corresponding to the assigned altitudes were taken from reference 11. As an aid to engine design, the values of the parameters within the rocket nozzle for 80, 90, 100, 110, and 120 percent of the throat pressure are tabulated in table IV. Equilibrium composition, γ_s, specific heat at constant pressure, coefficient of viscosity, coefficient of thermal conductivity, and mean molecular weight in the combustion chamber and at assigned exit temperatures are given in table V. The mole fraction of F_2 was always less than 0.00002 and therefore was not tabulated in table V.

Parameters. - The parameters are plotted in figures 2 to 7. Curves of specific impulse for the six altitudes are shown in figure 2 plotted against weight percent fuel. The maximum value of specific impulse for the sea-level curve is 364.6 pound-seconds per pound at 14.7 percent of fuel by weight.

The maximum values of specific impulse and the weight percents of fuel at which they occur were obtained by numerical differentiation of the calculated values and are shown in figure 3 as functions of altitude. The maximum specific impulse increased 19 percent for a change in altitude from sea level to 50,000 feet.

Curves of combustion-chamber temperature and nozzle-exit temperature for the six altitudes are presented in figure 4 as functions of weight percent fuel. The maximum combustion temperature obtained was 4581° K at 4.6 percent fuel by weight. The maximums of the exit temperature curves occur near the stoichiometric ratio.

Characteristic velocity and coefficient of thrust are plotted in figure 5 and ratios of the area at the nozzle exit to area at the throat are shown in figure 6 as functions of weight percent fuel.

Curves of mean molecular weight in the combustion chamber and nozzle exit are plotted against weight percent fuel in figure 7.

Frozen composition. - In order to compare data based on the assumptions of equilibrium and frozen composition during the expansion process, several additional calculations were made assuming frozen composition and are presented in the following table together with corresponding equilibrium data for two equivalence ratios and expansion to two altitudes:

Parameters	r = 1.0				r = 0.3			
	Sea level		50,000 feet		Sea level		50,000 feet	
	Equilibrium	Frozen	Equilibrium	Frozen	Equilibrium	Frozen	Equilibrium	Frozen
I, lb-sec/lb	341.5	312.8	420.7	363.4	364.6	351.7	430.7	411.0
c^*, ft/sec	7687	7288	7687	7288	8393	8167	8393	8167
C_F	1.429	1.381	1.761	1.604	1.398	1.386	1.651	1.619
S_e/S_t	3.987	3.049	21.30	12.25	3.384	3.154	14.24	13.00
T_e, $^\circ K$	3456	2074	2749	1112	1882	1597	1075	884
M_e	18.75	16.95	19.72	16.95	10.32	10.01	10.32	10.01

For a combustion pressure of 300 pounds per square inch absolute and an exit pressure of 1 atmosphere, the values of maximum specific impulse are 364.6 pound-seconds per pound at 14.7 percent fuel by weight for equilibrium composition during expansion and 356.2 pound-seconds per pound at 19.4 percent fuel by weight for frozen composition during expansion.

Chamber pressure effect. - Values of c^*, C_F, and S_e/S_t previously calculated at this laboratory for chamber pressures of 300, 1000, and 2000 pounds per square inch absolute at the stoichiometric equivalence ratio for an expansion ratio of 136.1 are given as follows:

P_c (lb/sq in. abs)	c^* (ft/sec)	C_F	S_e/S_t
300	7688	1.728	17.27
1000	7838	1.718	16.63
2000	7918	1.712	16.27

These parameters are very nearly linear with the logarithm of chamber pressure. Increasing chamber pressure by a factor of 2 results in changes of 1.0 percent for c^*, -0.4 percent for C_F, and -2.2 percent for S_e/S_t.

According to unpublished NACA data for liquid hydrazine with liquid fluorine, about the same percentage differences in these parameters due to chamber pressure were also found at the stoichiometric equivalence ratio and smaller differences were found on either side of stoichiometric.

It is expected that the values of c^*, C_F, and S_e/S_t given in this report for a chamber pressure of 300 pounds per square inch

absolute may be used for other chamber pressures at constant expansion ratios with similar small errors.

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TABLE I - PROPERTIES OF LIQUID PROPELLANTS

Propellant → / Properties ↓	Hydrogen	Fluorine
Molecular weight, M	2.016	38.00
Density, g/cc	[a]0.0709 (at -252.7° C)	[b]1.54 (at -196° C)
Freezing point, °C	[c]-259.20	[c]-217.96
Boiling point, °C	[c]-252.77	[c]-187.92
Viscosity, centipoises	[d]0.0215 (at -258.33° C)	--------
Enthalpy of formation at boiling point from elements at 25° C, ΔH_f, kcal/mole	[e]-1.895	[e]-3.030
Enthalpy of vaporization, ΔH, kcal/mole	[c]0.216 (at -252.77° C)	[c]1.51 (at -187.92° C)
Enthalpy of fusion, ΔH, kcal/mole	[c]0.028 (at -259.20° C)	[c]0.372 (at -217.96° C)

[a]Reference 3.
[b]Reference 4.
[c]Reference 2.
[d]Reference 5.
[e]Reference 1.

TABLE II - COEFFICIENT OF VISCOSITY

Temperature T °K	Viscosity μ, poise × 10^7						Temperature T °K	Viscosity μ, poise × 10^7					
	HF	H_2	N_2	F	H	N		HF	H_2	N_2	F	H	N
100	566	416	687	1,319	398	881	2600	13,971	3627	7,585	12,694	3449	9,634
200	1,120	683	1296	2,135	630	1466	2700	14,499	3716	7,772	13,032	3537	9,904
288	1,603	872	1731	2,751	802	1916	2800	15,028	3804	7,956	13,365	3623	10,172
300	1,669	896	1785	2,830	824	1974	2900	15,552	3891	8,138	13,696	3708	10,438
400	2,215	1083	2202	3,457	997	2438	3000	16,084	3977	8,318	14,022	3792	10,701
500	2,759	1251	2573	4,036	1156	2872							
600	3,301	1407	2914	4,582	1305	3284	3100	16,610	4069	8,496	14,345	3876	10,961
700	3,841	1554	3231	5,100	1445	3677	3200	17,138	4153	8,672	14,665	3958	11,220
800	4,381	1694	3530	5,596	1579	4056	3300	17,665	4237	8,846	14,982	4040	11,476
900	4,919	1828	3815	6,073	1707	4422	3400	18,192	4319	9,017	15,296	4121	11,730
1000	5,456	1956	4095	6,534	1831	4778	3500	18,718	4401	9,188	15,608	4201	11,982
1100	5,993	2080	4348	6,982	1950	5124	3600	19,244	4481	9,356	15,916	4280	12,233
1200	6,528	2200	4606	7,417	2066	5462	3700	19,770	4561	9,532	16,222	4358	12,481
1300	7,063	2316	4850	7,842	2178	5792	3800	20,296	4640	9,698	16,526	4436	12,728
1400	7,598	2433	5088	8,256	2288	6116	3900	20,822	4719	9,861	16,827	4513	12,973
1500	8,131	2543	5319	8,662	2395	6434	4000	21,347	4796	10,023	17,126	4589	13,216
1600	8,664	2652	5545	9,059	2500	6746	4100	21,871	4873	10,184	17,422	4665	13,458
1700	9,197	2757	5767	9,449	2602	7053	4200	22,395	4950	10,344	17,716	4740	13,698
1800	9,729	2861	5983	9,832	2703	7355	4300	22,921	5025	10,502	18,008	4815	13,937
1900	10,261	2962	6196	10,208	2802	7652	4400	23,445	5101	10,659	18,299	4889	14,174
2000	10,792	3062	6404	10,578	2898	7946	4500	23,970	5175	10,814	18,586	4962	14,410
2100	11,323	3160	6609	10,943	2994	8236	4600	24,492	5249	10,969	18,872	5035	14,644
2200	11,853	3256	6810	11,303	3088	8522	4700	25,016	5322	11,122	19,157	5107	14,877
2300	12,383	3351	7008	11,658	3180	8805	4800	25,540	5395	11,274	19,439	5179	15,109
2400	12,913	3444	7203	12,008	3271	9084	4900	26,064	5467	11,425	19,720	5250	15,339
2500	13,442	3536	7395	12,353	3361	9360	5000	26,587	5539	11,575	19,998	5321	15,568

TABLE III - CALCULATED PERFORMANCE OF LIQUID HYDROGEN WITH LIQUID FLUORINE

[Combustion-chamber pressure, 300 lb/sq in. absolute.]

Propellant			Combustion chamber		Characteristic velocity c^* (ft/sec)	Nozzle exit						
Equivalence ratio r	Weight-percent fuel	Density (g/cc)	Temperature T_c (^oK)	Mean molecular weight M_c		Altitude (ft)	Pressure P_e (atm)	Temperature T_e (^oK)	Mean molecular weight M_e	Ratio of nozzle-exit area to throat area S_e/S_t	Coefficient of thrust C_F	Specific impulse I (lb-sec/lb)
1.2	4.234	0.820	4568	17.95	7452	0	1.0	3290	19.68	3.861	1.424	329.8
						10,000	.6876	3085	19.78	5.003	1.491	345.4
						20,000	.4594	2834	19.83	6.581	1.555	360.2
						30,000	.2968	2555	19.85	8.830	1.615	374.2
						40,000	.1852	2265	19.85	12.13	1.672	387.2
						50,000	.1149	1999	19.84	16.77	1.720	398.4
1.1	4.601	0.788	4581	17.49	7571	0	1.0	3435	19.33	3.967	1.428	336.1
						10,000	.6876	3296	19.51	5.230	1.497	352.4
						20,000	.4594	3139	19.68	7.071	1.565	368.3
						30,000	.2968	2939	19.82	9.767	1.631	383.7
						40,000	.1852	2688	19.89	13.73	1.694	398.6
						50,000	.1149	2407	19.91	19.17	1.749	411.6
1.0	5.038	0.753	4573	16.95	7687	0	1.0	3456	18.75	3.987	1.429	341.5
						10,000	.6876	3336	18.94	5.281	1.499	358.2
						20,000	.4594	3207	19.14	7.192	1.568	374.5
						30,000	.2968	3069	19.35	10.10	1.635	390.6
						40,000	.1852	2915	19.55	14.63	1.701	406.4
						50,000	.1149	2749	19.72	21.30	1.761	420.7
0.9	5.567	0.715	4539	16.33	7801	0	1.0	3393	18.00	3.962	1.428	346.3
						10,000	.6876	3261	18.18	5.230	1.497	363.1
						20,000	.4594	3117	18.36	7.090	1.565	379.5
						30,000	.2968	2956	18.53	9.890	1.631	395.5
						40,000	.1852	2774	18.70	14.18	1.695	411.1
						50,000	.1149	2580	18.84	20.41	1.753	425.1
0.8	6.219	0.673	4470	15.61	7907	0	1.0	3252	17.14	3.892	1.425	350.1
						10,000	.6876	3108	17.29	5.117	1.493	366.8
						20,000	.4594	2953	17.45	6.905	1.559	383.0
						30,000	.2968	2783	17.61	9.589	1.623	398.8
						40,000	.1852	2596	17.76	13.69	1.685	414.1
						50,000	.1149	2400	17.87	19.62	1.741	427.8
0.7	7.045	0.626	4355	14.80	7997	0	1.0	3082	16.18	3.830	1.421	353.2
						10,000	.6876	2936	16.32	5.024	1.488	369.7
						20,000	.4594	2778	16.46	6.763	1.552	385.8
						30,000	.2968	2605	16.59	9.360	1.615	401.5
						40,000	.1852	2412	16.69	13.30	1.676	416.5
						50,000	.1149	2208	16.77	18.93	1.730	430.0

TABLE III - CALCULATED PERFORMANCE OF LIQUID HYDROGEN WITH LIQUID FLUORINE - Concluded

[Combustion-chamber pressure, 300 lb/sq in. absolute.]

Propellant			Combustion chamber		Characteristic velocity c^* (ft/sec)	Nozzle exit						
Equivalence ratio r	Weight-percent fuel	Density (g/cc)	Temperature T_c (^oK)	Mean molecular weight M_c		Altitude (ft)	Pressure P_e (atm)	Temperature T_e (^oK)	Mean molecular weight M_e	Ratio of nozzle-exit area to throat area S_e/S_t	Coefficient of thrust C_F	Specific impulse I (lb-sec/lb)
0.6	8.124	0.574	4187	13.87	8076	0	1.0	2894	15.10	3.786	1.418	356.1
						10,000	.6876	2745	15.21	4.953	1.484	372.6
						20,000	.4594	2581	15.31	6.639	1.548	388.6
						30,000	.2968	2396	15.40	9.126	1.610	404.0
						40,000	.1852	2191	15.46	12.85	1.668	418.8
						50,000	.1149	1980	15.49	18.11	1.721	431.9
0.5	9.593	0.515	3967	12.81	8163	0	1.0	2663	13.82	3.731	1.416	359.3
						10,000	.6876	2503	13.89	4.852	1.481	375.8
						20,000	.4594	2326	13.95	6.450	1.543	391.5
						30,000	.2968	2128	13.98	8.770	1.603	406.6
						40,000	.1852	1915	14.00	12.20	1.659	420.9
						50,000	.1149	1707	14.01	17.02	1.708	433.4
0.4	11.71	0.449	3693	11.56	8276	0	1.0	2339	12.25	3.610	1.411	362.8
						10,000	.6876	2164	12.27	4.644	1.473	378.9
						20,000	.4594	1977	12.29	6.098	1.532	394.1
						30,000	.2968	1782	12.30	8.203	1.588	408.5
						40,000	.1852	1584	12.30	11.31	1.640	421.9
						50,000	.1149	1401	12.30	15.69	1.686	433.6
0.3	15.03	0.374	3323	10.01	8393	0	1.0	1882	10.32	3.384	1.398	364.6
						10,000	.6876	1717	10.32	4.310	1.456	379.8
						20,000	.4594	1551	10.32	5.617	1.511	394.1
						30,000	.2968	1386	10.32	7.511	1.562	407.5
						40,000	.1852	1223	10.32	10.31	1.610	419.9
						50,000	.1149	1075	10.32	14.24	1.651	430.7
0.2	20.97	0.288	2720	7.969	8359	0	1.0	1338	8.013	3.155	1.383	359.4
						10,000	.6876	1211	8.013	3.998	1.437	373.4
						20,000	.4594	1086	8.013	5.183	1.488	386.6
						30,000	.2968	963	8.013	6.897	1.535	398.9
						40,000	.1852	845	8.013	9.428	1.579	410.2
						50,000	.1149	739	8.013	12.98	1.617	420.0

TABLE IV - CALCULATED PARAMETERS AT PRESSURES NEAR THROAT OF LIQUID HYDROGEN WITH LIQUID FLUORINE

[Combustion chamber pressure, 300 lb/sq in. absolute.]

Equivalence ratio r	Weight-percent fuel	$\frac{P_x}{P_t}$	Pressure P_x (atm)	Temperature T_x (^oK)	Mean molecular weight M_x	Ratio of nozzle-exit area to throat area S_x/S_t	Coefficient of thrust C_F	Specific impulse I (lb-sec/lb)
1.2	4.234	1.2	14.04	4402	18.21	1.0358	0.5467	126.6
		1.1	12.87	4363	18.26	1.0085	.6052	140.2
		1.0	11.70	4321	18.32	1.0000	.6627	153.5
		.9	10.53	4276	18.39	1.0080	.7202	166.8
		.8	9.363	4227	18.47	1.0326	.7786	180.3
1.1	4.601	1.2	14.07	4419	17.73	1.0359	0.5452	128.3
		1.1	12.89	4382	17.79	1.0085	.6038	142.1
		1.0	11.72	4342	17.85	1.0000	.6613	155.6
		.9	10.55	4298	17.92	1.0080	.7189	169.2
		.8	9.377	4250	18.00	1.0327	.7774	182.9
1.0	5.038	1.2	14.07	4412	17.18	1.0357	0.5447	130.1
		1.1	12.90	4376	17.24	1.0084	.6033	144.1
		1.0	11.73	4337	17.30	1.0000	.6609	157.9
		.9	10.55	4294	17.36	1.0081	.7185	171.7
		.8	9.381	4246	17.44	1.0328	.7770	185.7
0.9	5.567	1.2	14.07	4377	16.55	1.0356	0.5451	132.2
		1.1	12.89	4341	16.60	1.0084	.6037	146.4
		1.0	11.72	4301	16.66	1.0000	.6613	160.3
		.9	10.55	4258	16.72	1.0081	.7188	174.3
		.8	9.377	4210	16.79	1.0327	.7774	188.5
0.8	6.219	1.2	14.04	4303	15.82	1.0353	0.5469	134.4
		1.1	12.87	4266	15.87	1.0083	.6054	148.8
		1.0	11.70	4225	15.92	1.0000	.6629	162.9
		.9	10.53	4180	15.98	1.0080	.7203	177.0
		.8	9.360	4130	16.04	1.0325	.7787	191.4
0.7	7.045	1.2	14.00	4180	14.99	1.0348	0.5502	136.8
		1.1	12.83	4141	15.03	1.0082	.6085	151.2
		1.0	11.66	4098	15.08	1.0000	.6658	165.5
		.9	10.50	4051	15.13	1.0079	.7231	179.7
		.8	9.330	3999	15.19	1.0321	.7813	194.2

TABLE IV - CALCULATED PARAMETERS AT PRESSURES NEAR THROAT OF LIQUID HYDROGEN WITH LIQUID FLUORINE - Concluded

[Combustion-chamber pressure, 300 lb/sq in. absolute.]

Equivalence ratio r	Weight-percent fuel	$\frac{P_x}{P_t}$	Pressure P_x (atm)	Temperature T_x (^oK)	Mean molecular weight M_x	Ratio of nozzle-exit area to throat area S_x/S_t	Coefficient of thrust C_F	Specific impulse I (lb-sec/lb)
0.6	8.124	1.2	13.95	4006	14.04	1.0344	0.5537	139.0
		1.1	12.79	3965	14.08	1.0081	.6117	153.6
		1.0	11.63	3921	14.12	1.0000	.6689	167.9
		.9	10.46	3873	14.17	1.0078	.7260	182.2
		.8	9.300	3820	14.22	1.0317	.7839	196.8
0.5	9.593	1.2	13.92	3786	12.96	1.0341	0.5561	141.1
		1.1	12.76	3745	13.00	1.0081	.6140	155.8
		1.0	11.60	3701	13.03	1.0000	.6710	170.2
		.9	10.44	3654	13.08	1.0078	.7280	184.7
		.8	9.281	3601	13.12	1.0316	.7858	199.4
0.4	11.71	1.2	13.88	3512	11.68	1.0337	0.5588	143.7
		1.1	12.73	3472	11.71	1.0080	.6165	158.6
		1.0	11.57	3429	11.74	1.0000	.6733	173.2
		.9	10.41	3381	11.77	1.0077	.7302	187.8
		.8	9.256	3328	11.81	1.0312	.7879	202.7
0.3	15.03	1.2	13.78	3135	10.09	1.0324	0.5666	147.8
		1.1	12.63	3094	10.10	1.0077	.6239	162.8
		1.0	11.48	3048	10.12	1.0000	.6803	177.5
		.9	10.33	2998	10.14	1.0074	.7367	192.2
		.8	9.185	2942	10.16	1.0299	.7939	207.1
0.2	20.97	1.2	13.45	2502	7.991	1.0292	0.5914	153.7
		1.1	12.33	2457	7.994	1.0070	.6472	168.2
		1.0	11.21	2407	7.997	1.0000	.7022	182.5
		.9	10.09	2353	8.001	1.0068	.7573	196.7
		.8	8.965	2293	8.004	1.0277	.8130	211.2

TABLE V - PROPERTIES AND COMPOSITION IN COMBUSTION CHAMBER AND FOLLOWING AN ISENTROPIC EXPANSION TO ASSIGNED EXIT TEMPERATURES FOR LIQUID HYDROGEN WITH LIQUID FLUORINE

[Combustion-chamber pressure, 300 lb/sq in. absolute]

Temperature T (°K)	Pressure P (atm)	$\gamma_s \left(\frac{\partial \log P}{\partial \log \rho}\right)_s$	Specific heat at constant pressure c_p (cal/(g)(°K))	Coefficient of viscosity μ (poise $\times 10^7$)	Coefficient of thermal conductivity k [(cal/(sec)(cm)(°K)) $\times 10^6$]	Molecular weight M	Equilibrium composition (mole fraction)			
							HF	H_2	F	H
r = 1.2 (4.23 percent fuel by weight)										
4568	20.41	1.1605	2.1944	22061	5146	17.955	0.64882	0.01029	0.25613	0.08474
4300	11.16	1.1560	2.0528	21152	4628	18.356	.68858	.00758	.23658	.06725
3200	.8425	1.2225	0.6673	16659	1321	19.736	.82345	.00027	.17130	.00498
3000	.5970	1.2720	0.5039	15707	988	19.806	.83012	.00007	.16816	.00165
2000	.1151	1.3528	0.3841	10758	548	19.840	.83334	.00000	.16665	.00000
r = 1.1 (4.60 percent fuel by weight)										
4581	20.41	1.1574	2.4300	22170	5703	17.487	0.66055	0.01565	0.21744	0.10635
4300	10.60	1.1513	2.3337	21226	5248	17.916	.70497	.01236	.19457	.08809
3400	.9202	1.1506	1.4312	17707	2761	19.363	.85339	.00267	.11883	.02510
3200	.5343	1.1662	1.0793	16802	2026	19.620	.87946	.00126	.10568	.01360
2500	.1340	1.3145	0.4201	13340	727	19.914	.90882	.00000	.09104	.00013
r = 1.0 (5.04 percent fuel by weight)										
4573	20.41	1.1560	2.5687	22169	6019	16.947	0.67037	0.02364	0.17662	0.12936
4300	10.72	1.1497	2.4952	21251	5607	17.354	.71443	.02031	.15294	.11232
3300	.6149	1.1403	1.6641	17304	3106	18.999	.89086	.00826	.05870	.04218
3200	.4488	1.1423	1.5295	16853	2796	19.155	.90754	.00719	.04982	.03544
2600	.07730	1.1946	0.7422	13934	1209	19.838	.98085	.00214	.01064	.00636
r = 0.9 (5.57 percent fuel by weight)										
4539	20.41	1.1566	2.5809	22011	6016	16.326	0.67693	0.03604	0.13447	0.15256
4300	11.70	1.1516	2.4917	21196	5597	16.658	.71424	.03366	.11371	.13840
3400	1.024	1.1486	1.6447	17628	3143	17.991	.85933	.02834	.03483	.07750
3200	.5786	1.1564	1.3662	16712	2511	18.257	.88625	.02944	.02118	.06314
2500	.09488	1.2221	0.6799	13230	1074	18.888	.93773	.04413	.00104	.01710
r = 0.8 (6.22 percent fuel by weight)										
4470	20.41	1.1598	2.4580	21619	5658	15.612	0.67761	0.05618	0.09296	0.17325
4200	11.04	1.1562	2.2954	20654	5062	15.950	.71565	.05570	.07162	.15703
3100	.6733	1.1732	1.2316	15943	2192	17.303	.84647	.07517	.00759	.07077
3000	.5193	1.1778	1.1450	15460	1991	17.407	.85385	.07932	.00534	.06149
2300	.09092	1.2436	0.6304	11980	921	17.914	.88406	.10540	.00015	.01039
r = 0.7 (7.05 percent fuel by weight)										
4355	20.41	1.1666	2.2540	20877	5056	14.796	0.66793	0.09007	0.05595	0.18605
4100	11.71	1.1653	2.0826	19911	4475	15.074	.69698	.09403	.04049	.16850
2900	.6274	1.1846	1.1403	14607	1888	16.352	.79797	.14493	.00205	.05505
2800	.4855	1.1899	1.0573	14133	1708	16.440	.80294	.15033	.00135	.04538
2100	.08977	1.2756	0.5760	10736	777	16.796	.82172	.17393	.00002	.00433

TABLE V - PROPERTIES AND COMPOSITION IN COMBUSTION CHAMBER AND FOLLOWING AN ISENTROPIC EXPANSION TO ASSIGNED EXIT TEMPERATURES FOR LIQUID HYDROGEN WITH LIQUID FLUORINE - Concluded

[Combustion-chamber pressure, 300 lb/sq in. absolute]

Temperature T (°K)	Pressure P (atm)	$\gamma_s \left(\frac{\partial \log P}{\partial \log \rho}\right)_s$	Specific heat at constant pressure c_p (cal/(g)(°K))	Coefficient of viscosity μ (poise $\times 10^7$)	Coefficient of thermal conductivity k $\left\{\frac{cal}{(sec)(cm)(°K)}\right\}$ $\times 10^6$	Molecular weight M	Equilibrium composition (mole fraction)			
							HF	H_2	F	H
r = 0.6 (8.12 percent fuel by weight)										
4187	20.41	1.1752	2.0844	19680	4455	13.870	0.64244	0.14610	0.02826	0.18319
3900	11.10	1.1744	1.9207	18539	3886	14.145	.66615	.15777	.01782	.15826
2800	.7898	1.1931	1.1164	13702	1754	15.168	.73265	.22320	.00079	.04335
2700	.6152	1.2000	1.0287	13242	1578	15.238	.73636	.22861	.00051	.03453
1900	.09595	1.3074	0.5545	9500	679	15.500	.74952	.24920	.00000	.00128
r = 0.5 (9.59 percent fuel by weight)										
3967	20.41	1.1818	1.9984	18023	3951	12.812	0.59784	0.23099	0.01177	0.15940
3700	11.57	1.1807	1.8561	16954	3470	13.036	.61325	.24760	.00703	.13212
2600	.8624	1.2158	0.9874	12286	1433	13.847	.65869	.31790	.00018	.02323
2400	.5428	1.2394	0.8330	11406	1154	13.924	.66250	.32517	.00006	.01228
1700	.1131	1.3304	0.5733	8271	621	14.009	.66658	.33317	.00000	.00025
r = 0.4 (11.71 percent fuel by weight)										
3693	20.41	1.1872	1.9132	15931	3390	11.561	0.53340	0.34681	0.00380	0.11599
3400	10.86	1.1882	1.7165	14811	2855	11.758	.54452	.36787	.00188	.08573
2400	1.142	1.2501	0.8928	10821	1186	12.238	.56864	.42170	.00003	.00964
2100	.5992	1.2900	0.7424	9583	905	12.281	.57068	.42670	.00000	.00262
1400	.1147	1.3530	0.6194	6612	543	12.297	.57143	.42856	.00000	.00001
r = 0.3 (15.03 percent fuel by weight)										
3323	20.41	1.2004	1.7094	13313	2606	10.007	0.44681	0.49251	0.00073	0.05995
3100	12.80	1.2073	1.5251	12534	2220	10.101	.45138	.50615	.00036	.04211
2100	1.596	1.2977	0.8584	8903	979	10.311	.46112	.53708	.00000	.00180
1700	.6606	1.3326	0.7730	7383	748	10.319	.46151	.53836	.00000	.00013
1200	.1725	1.3667	0.7176	5409	518	10.320	.46154	.53846	.00000	.00000
r = 0.2 (20.97 percent fuel by weight)										
2720	20.41	1.2518	1.3631	9811	1643	7.969	0.33148	0.65754	0.00003	0.01096
2500	13.40	1.2704	1.2328	9144	1412	7.991	.33240	.66203	.00001	.00557
1500	1.553	1.3465	0.9639	5942	757	8.013	.33333	.66666	.00000	.00001
1200	.6648	1.3667	0.9242	4909	606	8.013	.33333	.66667	.00000	.00000
800	.1524	1.3908	0.8827	3460	413	8.013	.33333	.66667	.00000	.00000

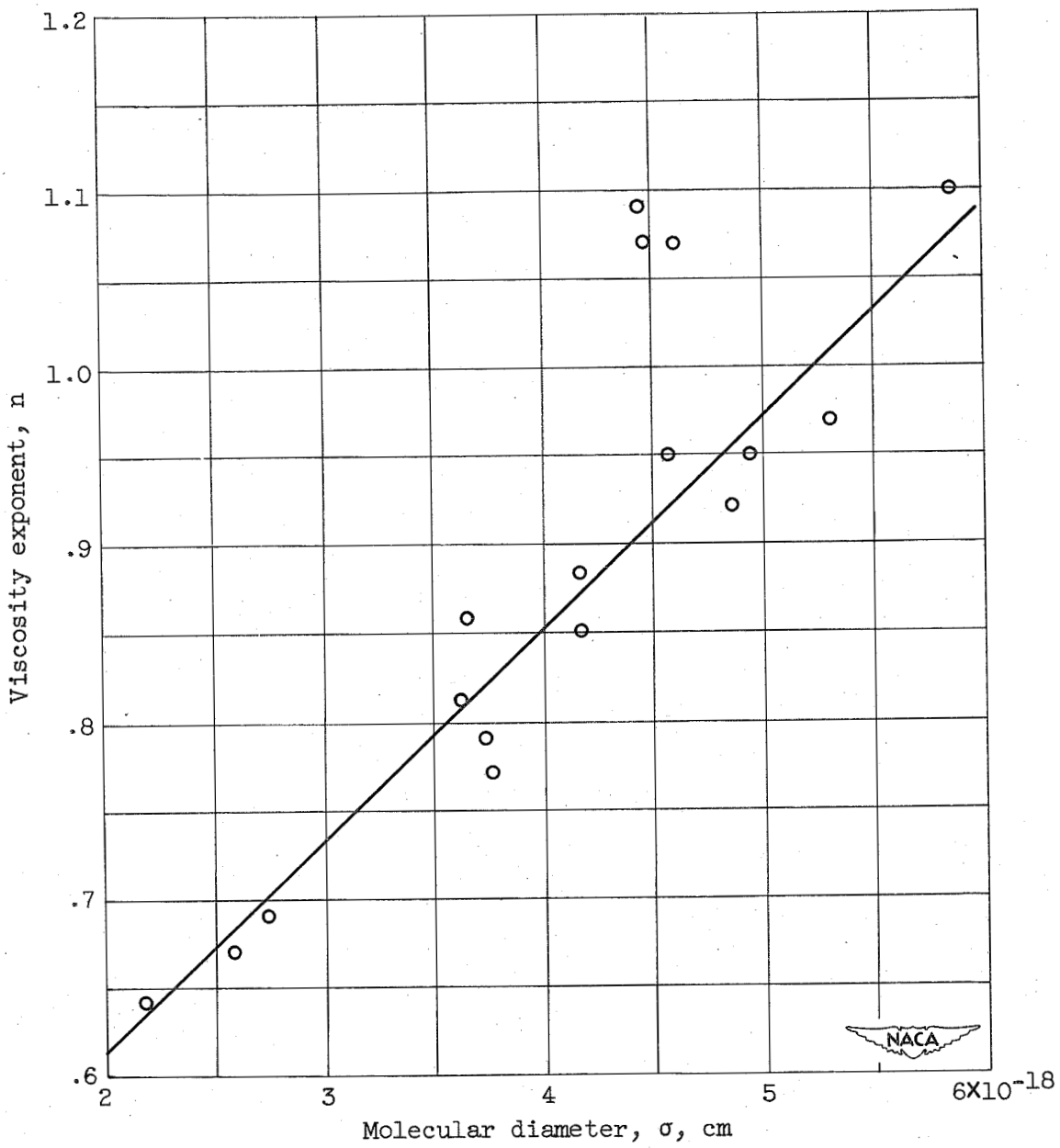

Figure 1. - Exponent n for equation $\mu = \mu_{288}\left(\frac{T}{288}\right)^n$. (Fig. 1 of reference 7)

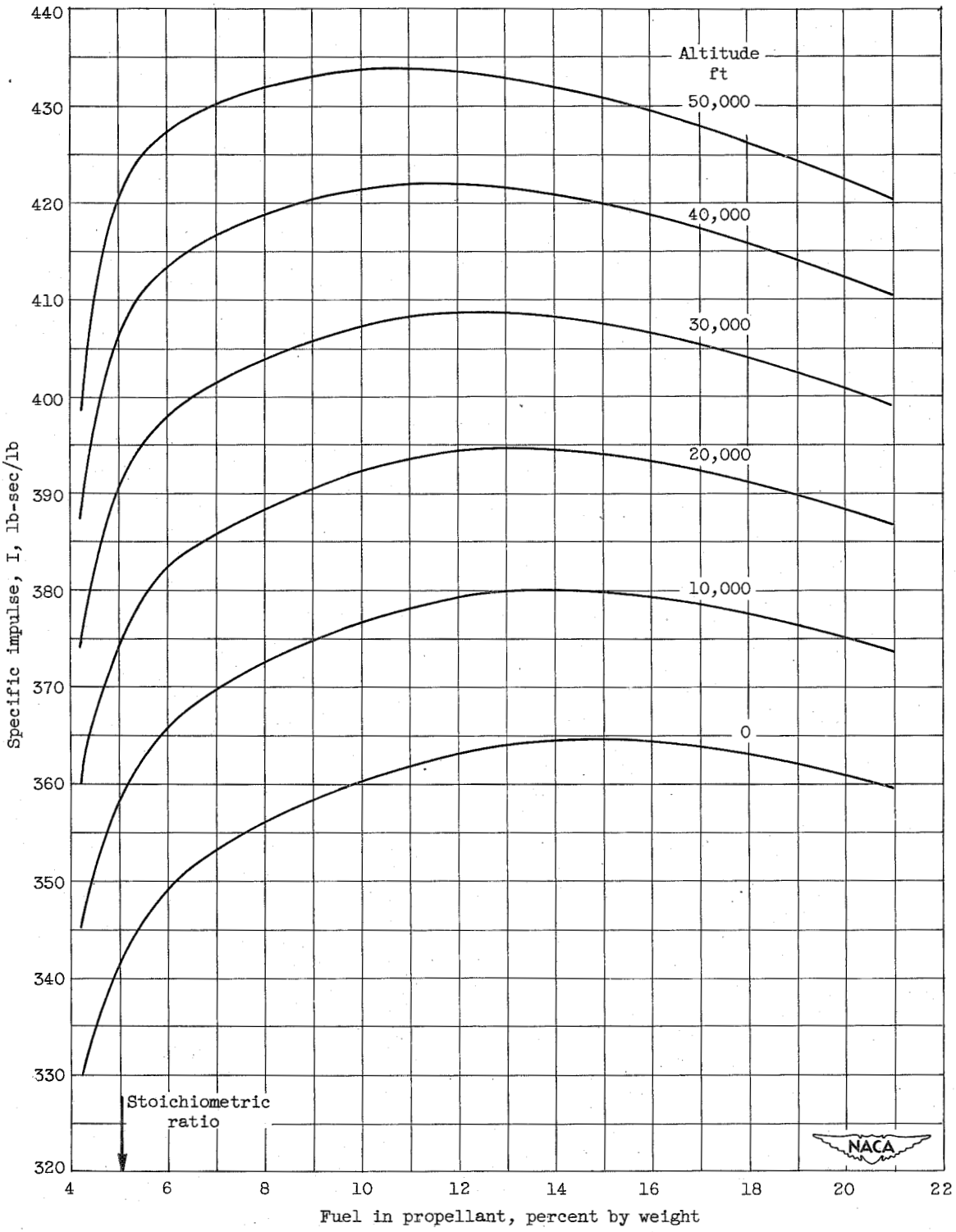

Figure 2. - Theoretical specific impulse of liquid hydrogen with liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.

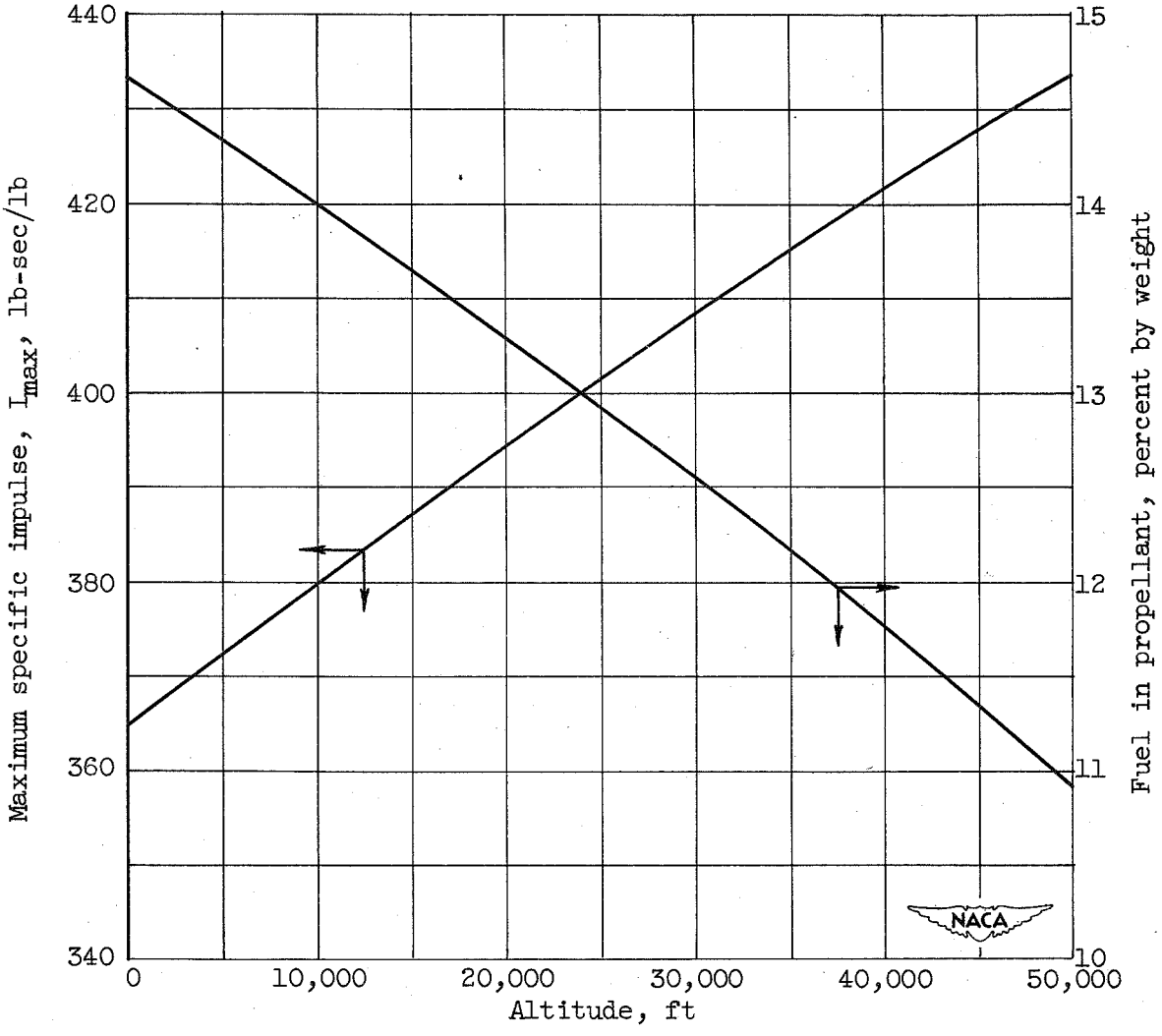

Figure 3. - Maximum theoretical specific impulse and corresponding weight percent of fuel in propellant of liquid hydrogen with liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.

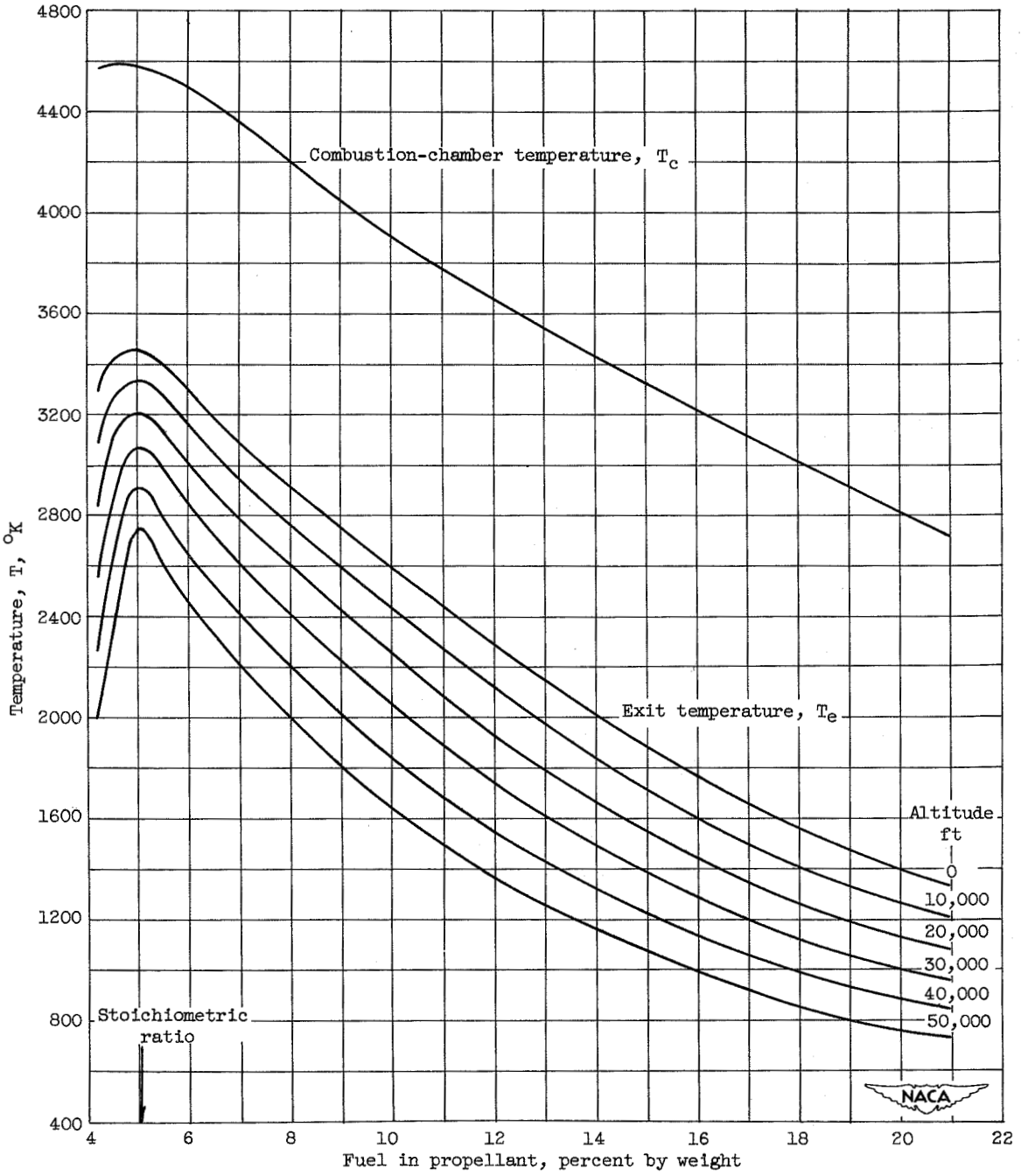

Figure 4. - Theoretical combustion-chamber temperature and nozzle-exit temperature of liquid hydrogen with liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.

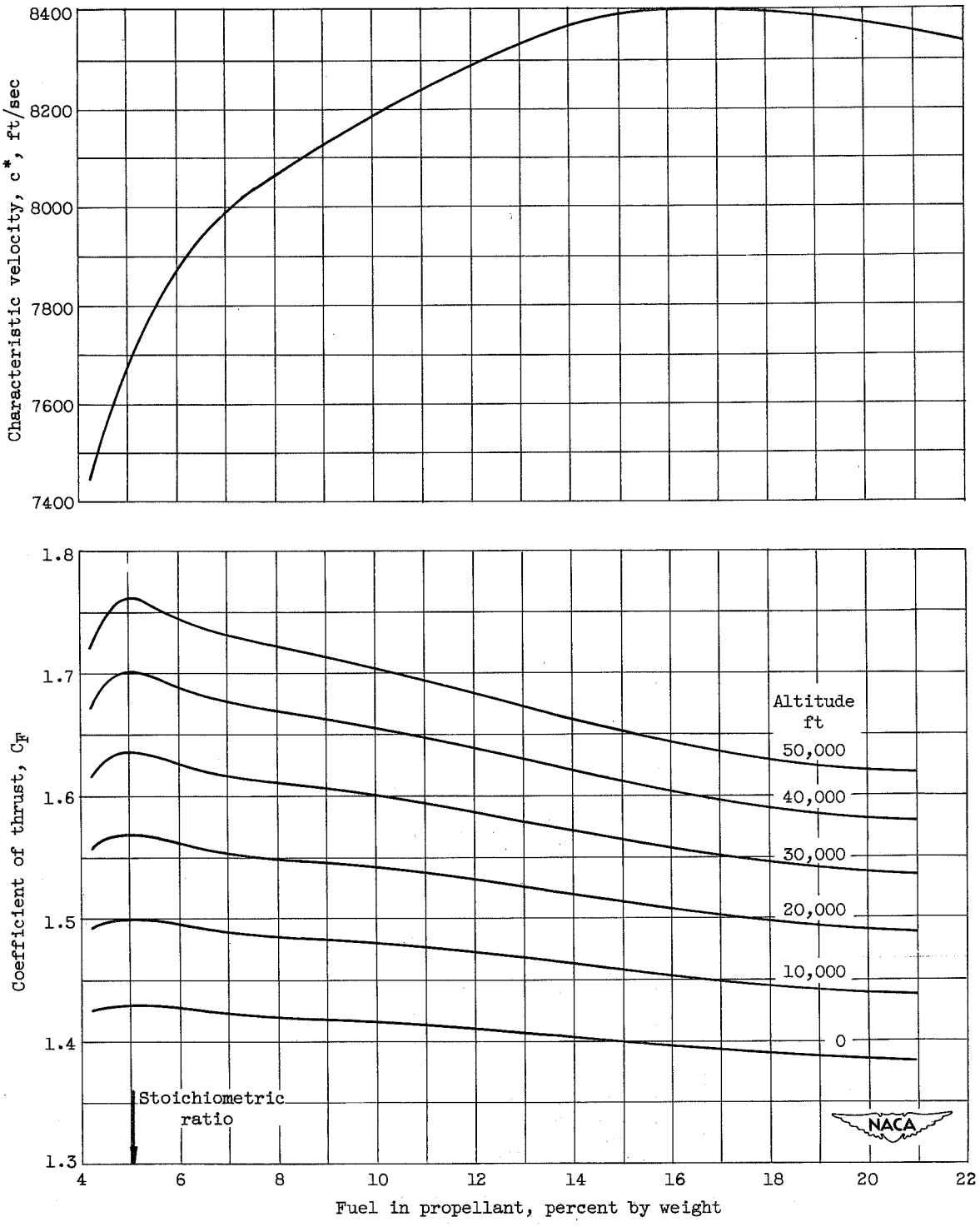

Figure 5. - Theoretical characteristic velocity and coefficient of thrust of liquid hydrogen and liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.

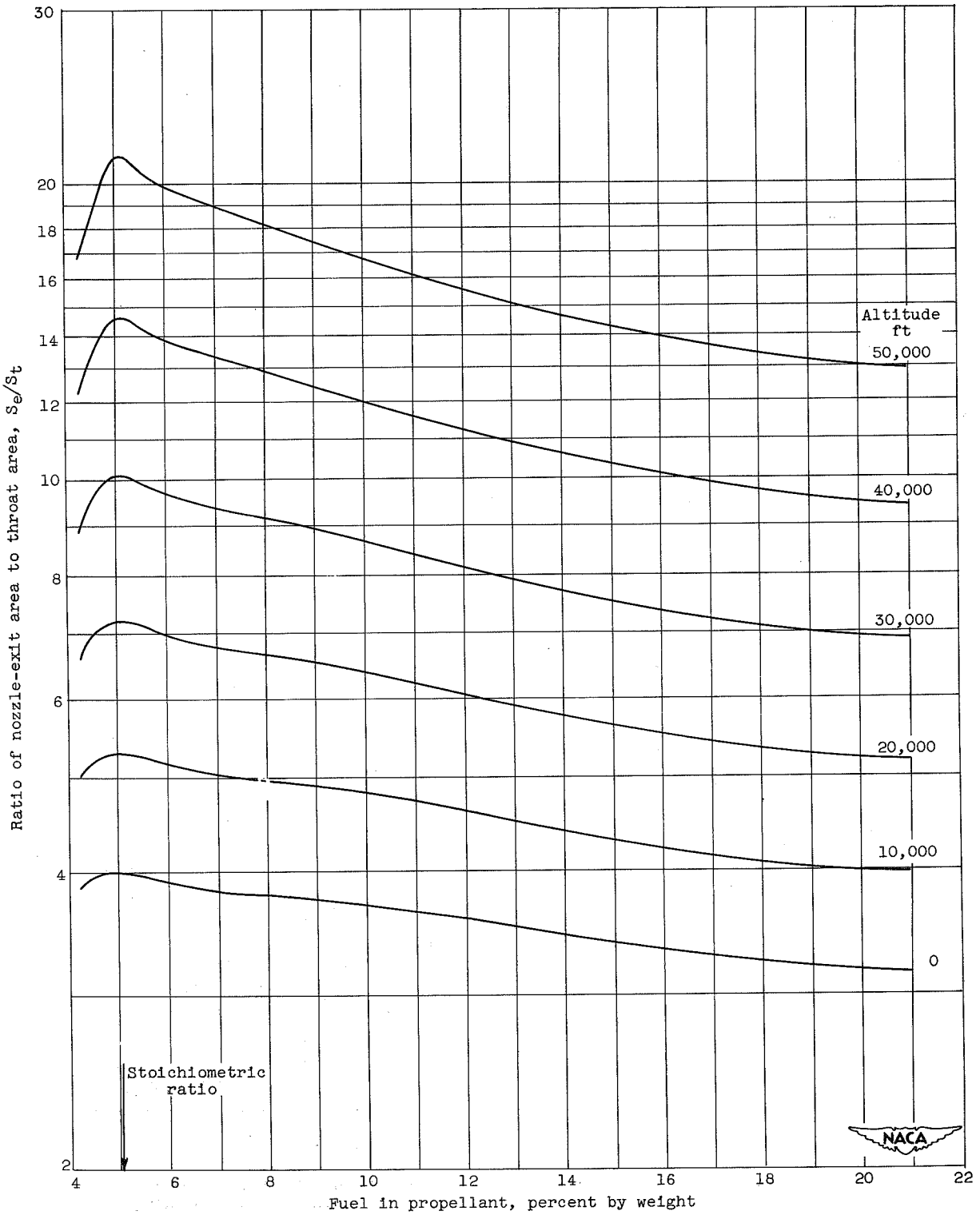

Figure 6. - Theoretical ratios of nozzle-exit area to throat area of liquid hydrogen with liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.

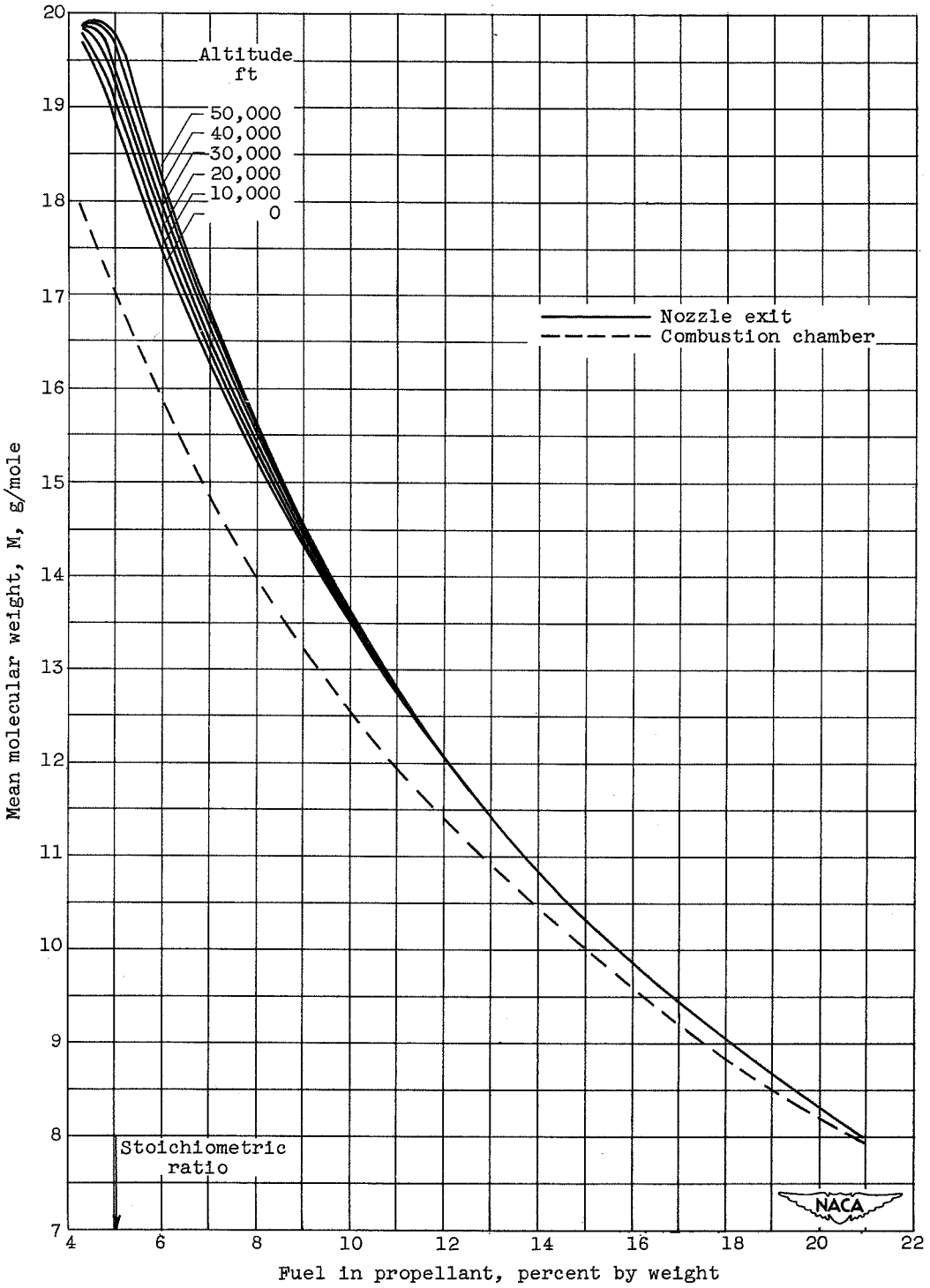

Figure 7. - Theoretical mean molecular weight in combustion chamber and at nozzle exit of liquid hydrogen with liquid fluorine. Isentropic expansion assuming equilibrium composition; combustion-chamber pressure, 300 pounds per square inch absolute; exit pressure corresponding to altitude indicated.